Physics I
ISI B.Math
Midterm Exam : March 1, 2013

Total Marks: 60

Answer question 1 and any 4 from the rest

1.(**Marks = $\mathbf{3 \times 4 = 12}$**)

(i)Suppose that a reference frame fixed to the earth is inertial. Which of the following are then inertial frames ? A frame fixed to to a car which is

(a) Moving with constant speed around a circular race track
(b) Moving with constant speed along a straight undulating road.
(c) Moving with constant speed up an inclined plane with constant gradient.
(d)Going down a hill with its engine turned off.
(e) None of the above.

(ii)A particle is moving under the influence of a force $\mathbf{F} = (\frac{a \sin t}{r^2} - \frac{b \cos t}{r^3})\hat{\mathbf{r}}$, where a and b are constants. Which of the following statements are not true about the motion of the particle ?

(a) Angular momentum is not conserved
(b) Total mechanical energy is conserved
(c) $\nabla \times \mathbf{F} = 0$.
(d) The work done by the particle in moving from one point to another is independent of path.
(e) The motion remains confined to a plane.

(iii) An undamped harmonic oscillator of mass m and angular frequency ω_0 moves in one dimension along the x- axis. If we plot x vs the linear momentum p_x (a phase space plot) for a given set of initial conditions, the resulting curve will be
(a) closed
(b) open
(c) can be closed or open depending upon the initial conditions.

(iv) A particle P of mass m can move under the gravitational attraction of two particles of equal mass M, fixed at the points $(0, 0, \pm a)$. The origin is a position of
(a) stable equilibrium
(b) unstable equilibrium
(c) it is not possible to determine whether O is a stable or unstable equilibrium point from the given information.

2.(**Marks = $\mathbf{3 + 4 + 3 + 2 = 12}$**)

The luckless Daniel (D) is thrown into a circular arena of radius a containing a lion (L). Initially the lion is at the centre (O) of the arena while Daniel is at the perimeter. Daniel's strategy is to run with his maximum speed u around the perimeter. The lion responds by running at his maximum speed U is such a way that it remains on the (moving) radius OD.

(i)Show that r, the distance of L from O, satisfies the differential equation

$$\dot{r}^2 = \frac{u^2}{a^2}\left(\frac{U^2a^2}{u^2} - r^2\right)$$

(ii)Find r as a function of t.

(iii) If $U \geq u$, show that Daniel will be caught, and find out how long this will take.

(iv)Show that the path taken by the lion is an arc of a circle.

3. **Marks = 6 + 6 = 12**

A communications satellite is in circular orbit around the earth at a radius R with a speed v. A rocket accidentally fires quite suddenly, giving the rocket an outward radial velocity of magnitude v in addition to its original velocity.

(i) Calculate the ratio of the new energy and the angular momentum to the old.

(ii) Describe the subsequent motion of the satellite and plot the kinetic energy $T(r)$,potential energy $U(r)$, effective potential energy U_{eff} and total energy $E(r)$ after the rocket fires.

4.(**Marks = 6 + 6 = 12**):

A particle of mass m moves in one dimension under a conservative force with potential energy

$$V(x) = \frac{cx}{x^2 + a^2}$$

where $a, c \geq 0$.

(i) Find the position of stable equilibrium and the period of small oscillations about it.

(ii) If the particle starts from this point with velocity v, find the range of values of v for which it (a) oscillates (b) escapes to $-\infty$ (c) escapes to $+\infty$.

5. (**Marks = 6 + 6 = 12**)

(i) A mass m is attached to a spring of spring constant k. The whole arrangement is immersed in oil which exerts a damping force on the mass of the form $F_d = -2\sqrt{mk}\dot{x}$. It starts out at a position $x_0 > 0$. What is the maximum initial speed (directed towards the origin) it can have and not cross the origin ?

(ii)The mass-spring arrangement is now taken out of the oil and a driving force $F(t) = F_0 \sin(\omega t)$ is switched on at $t = 0$. Find the displacement $x(t)$ of the mass at $t > 0$, if $x = 0$ and $v = 0$ at $t = 0$. What is the value of ω for resonance ?

6. (**Marks = 12**)

An electron of mass m and charge $-e$ is moving under the combined influence of a uniform electric field $E_0\mathbf{j}$ and a uniform magnetic field $B_0\mathbf{k}$. [Recall that the Lorentz force on a particle with charge q is given by $\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$] . Initially the electron is at the origin and is moving with velocity $u\mathbf{i}$. Show that the trajectory of the electron is given by

$$x = a(\Omega t) + b \sin \Omega t,\ y = b(1 - \cos \Omega t),\ z = 0$$

where $\Omega = \frac{eB_0}{m}$, $a = \frac{E_0}{\Omega B_0}$ and $b = \frac{(uB_0 - E_0)}{\Omega B_0}$.